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CONTROLLING THE FREE-SURFACE PROFILE OF FILM FLOW OVER COMPLEX TOPOGRAPHY

A. S. Ovcharova

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A two-dimensional model describing nonisothermal viscous thin film flow over complex topography is considered. The model is based on the Navier–Stokes equations in the Oberbeck–Boussinesq approximation. A numerical analysis of the effect of thermal loading on the location of the film free surface is performed. It is shown that changing the substrate temperature function, it is possible to control the free-surface profile on separate topographical features. The results of solution of model problems are presented).

Key words: *thin film, viscous liquid, free surface, Marangoni effect.*

Introduction. In viscous thin film flow over complex topography, so-called capillary ridges can develop due to topographic roughness and fluid surface tension. Kalliadasis et al. [1] studied the formation of the ridges in the lubrication approximation for an isothermal flow over a trench. They showed that the location of the free surface depends on three dimensionless parameters: the height of the trench edges, their slope to the base, and the width of the trench. The studies were performed for a wide range of these parameters. Mazouchi and Homsy [2] considered this problem in the Stokes approximation.

In the present paper, we study nonisothermal viscous thin film flow over a trench using the Navier–Stokes equations in the Oberbeck–Boussinesq approximation. The model problems were solved using the data given in [2].

Mathematical Model. We consider the motion of a viscous incompressible fluid with density ρ, kinematic viscosity ν, and surface tension $\sigma(T)$ in a constant gravitational field over complex topography [see Fig. 1, where $f_0(x)$ is the substrate surface, $f = f(t,x)$ is the free surface, and D is the height of the trench edges; and W is the width of the trench]. The fluid flow and heat transfer are described by the Oberbeck–Boussinesq equations, which are written in the variables ψ, ω, and θ as follows:

$$\frac{\partial\omega}{\partial t} + \frac{\partial}{\partial x}\Big(\omega\,\frac{\partial\psi}{\partial y}\Big) - \frac{\partial}{\partial y}\Big(\omega\,\frac{\partial\psi}{\partial x}\Big) = \frac{1}{\mathrm{Re}}\,\Delta\omega + \frac{\mathrm{Gr}}{\mathrm{Re}^2}\,\frac{\partial\theta}{\partial y}; \tag{1}$$

$$\Delta\psi + \omega = 0; \tag{2}$$

$$\frac{\partial\theta}{\partial t} + \frac{\partial}{\partial x}\Big(\theta\,\frac{\partial\psi}{\partial y}\Big) - \frac{\partial}{\partial y}\Big(\theta\,\frac{\partial\psi}{\partial x}\Big) = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\,\Delta\theta. \tag{3}$$

Here Re, Gr, and Pr are the Reynolds, Grashof, and Prandtl numbers, respectively, and $\theta = (T - T_0)/\Delta T$ (T_0 is the characteristic temperature and ΔT is the temperature gradient); the pressure scale is the quantity ρv_0^2 (v_0 is the characteristic velocity). The gravitational vector $\boldsymbol{g}$ is parallel to the x axis and coincides with its positive direction.

The stream function is given by the relations

$$u = \frac{\partial\psi}{\partial y}, \qquad v = -\frac{\partial\psi}{\partial x}.$$

Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090.
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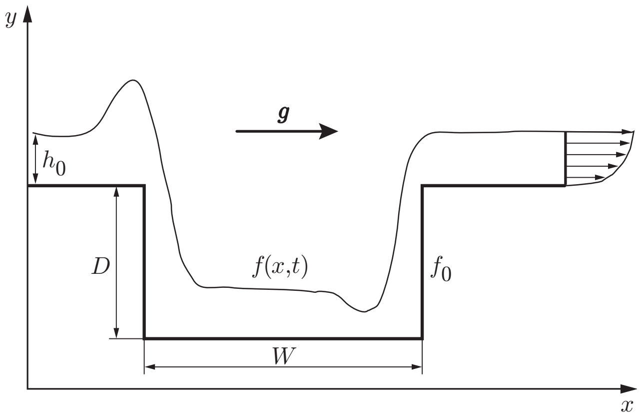

Fig. 1

The initial conditions are assumed to be specified. The boundary conditions correspond to the no-slip condition on the substrate surface $f_0(x)$:

$$\psi = 0, \qquad \frac{\partial \psi}{\partial n} = 0.$$

At the entrance to and exit from the solution region, the stream function is specified so as to ensure a semiparabolic velocity profile in the section perpendicular to the substrate surface. At the entrance, $\omega = 0$, and at the exit, $\partial\omega/\partial x = 0$.

To find the free surface $f(x,t)$, we determine the normal and tangential vectors to it at each point:

$$\boldsymbol{n} = \Big\{\frac{-f_x}{\sqrt{1+f_x^2}}, \frac{1}{\sqrt{1+f_x^2}}\Big\}, \qquad \boldsymbol{s} = \Big\{\frac{1}{\sqrt{1+f_x^2}}, \frac{f_x}{\sqrt{1+f_x^2}}\Big\}.$$

Next, we assume that the surface tension $\sigma(T)$ is a linear function of temperature:

$$\sigma(T) = \sigma_0(1-\sigma_T(T-T_0)), \quad \sigma_0 = \sigma(T_0), \quad \sigma_T = \frac{1}{\sigma_0}\frac{d\sigma}{dT}\Big|_{T=T_0}, \quad \sigma_0 > 0, \quad \sigma_T > 0.$$

Then, the free surface can be defined by the equation

$$f_t + \sqrt{1+f_x^2}\,\frac{\partial \psi}{\partial s} = 0,$$

and the boundary conditions for ψ and ω can be written in explicit form

$$\frac{\partial \psi}{\partial n} = v_s, \qquad \omega = 2\Big(\frac{v_s}{R} + \frac{\partial v_n}{\partial s}\Big) + \frac{\mathrm{Mn}}{\mathrm{Re}}\frac{\partial \theta}{\partial s},$$

where v_s is a solution of the equation

$$\frac{\partial v_s}{\partial t} + v_s\frac{\partial v_s}{\partial s} = \frac{2}{\mathrm{Re}}\frac{\partial^2 v_s}{\partial s^2} + F; \tag{4}$$

$$F = -\frac{1}{\mathrm{Re}}\frac{\partial \omega}{\partial n} + \frac{\mathrm{Ca}^{-1}}{\mathrm{Re}}\frac{\partial}{\partial s}\Big[\frac{1}{R}\Big(1 - \frac{\sigma_T\Delta T}{\sigma_0}\,\theta\Big)\Big] + \frac{1}{\sqrt{1+f_x^2}}(-\mathrm{Gr}\theta + \mathrm{G}) - 2\frac{\partial}{\partial s}\frac{v_n}{R} + v_n\omega + f_x\frac{v_n^2}{R};$$

$$v_n = f_t/\sqrt{1+f_x^2}.$$

Here v_s and v_n are the tangential and normal velocity components for the points of the free surface, R is the curvature radius of the surface $f(x,t)$, $\mathrm{Ca} = \rho_0 v_0 \nu/\sigma_0$ is the capillary number, $\mathrm{G} = g_0 h_0/v_0^2$ is the Galileo number, g_0 is the acceleration due to gravity, h_0 is the characteristic film width, and $\mathrm{Mn} = \sigma_T\Delta T/(\rho_0 v_0 \nu)$ is the Marangoni number. For the steady-state case, Eq. (4) is derived in [3].

Method of Solution. For the boundary conditions at the free surface specified above, problem (1)–(3) can be solved with the use of any methods commonly employed to solve heat- and mass-transfer problem in closed

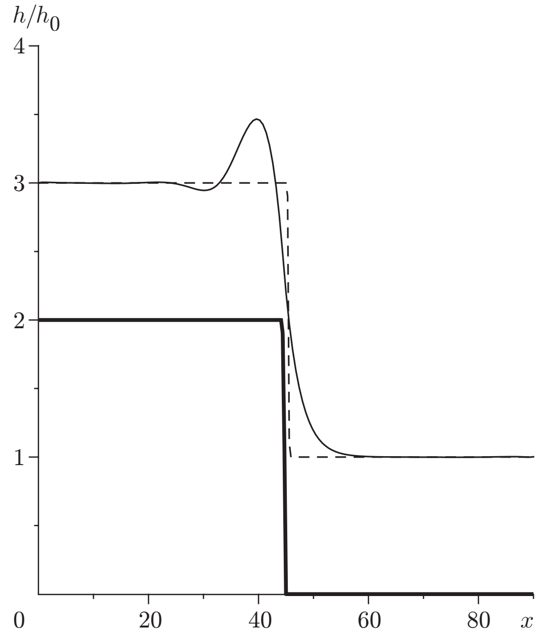

Fig. 2

domains in the variables ψ, ω, and θ. We note that in the Oberbeck–Boussinesq model, Eqs. (1)–(3) are of the same type and can be solved using the same computational procedure.

In the present study, we use the method of solution in a regular domain. The domain occupied by the fluid is mapped onto a rectangle with sides $0 \leqslant \xi \leqslant L$ and $0 \leqslant \eta \leqslant 1$ by the transformation

$$x = \xi, \qquad y = f_0(x) + \eta(f(x,t) - f_0(x)).$$

Then, all boundaries of the domain, including the free surface, lie on the coordinate axes. The method of solving the heat- and mass-transfer equations (1)–(3) resulting from such transformation is described in detail in [3].

Numerical Calculations and Discussion of Results. We consider a film flow in a constant gravitational field ($g_x = 1$ and $g_y = 0$) for $\mathrm{G} = 1$ and $\mathrm{Re} = 1$. The dimensionless height of the trench edges is equal to 2. The slope of the edges to the horizontal is approximately 80°. The capillary number Ca is in the range of 0.01–0.10.

We first consider an isothermal fluid flow ($\mathrm{Gr} = 0$ and $\mathrm{Mn} = 0$). Figure 2 gives the calculation for a film flow down a substrate surface (a step down) for $\mathrm{Ca} = 0.025$. The dashed curve shows the initial position of the free surface, and the solid curve is the steady-state solution. The film motion develops as follows. Under the action of surface tension, a capillary wave forms upstream (of the corner), which propagates against the flow until the motion becomes steady-state. Downstream, the free surface enters a steady-state (initial) level exponentially. Figure 3 shows the steady-state solution for a step up topography. In this case, too, an upstream propagating capillary wave gradually develops (the free-surface level decreases).

Figure 4 gives the results of the steady-state solution for a complex topography which is a combination of a step down and a step up separated by a distance of $W = 40$. The calculations were performed for capillary numbers $\mathrm{Ca} = 0.10$, 0.05, 0.025, and 0.01. The effect of the capillary number on the behavior of the free surface is apparent. The smaller the value of Ca, the larger and wider the capillary ridge above the upper corner of the substrate and the larger the depression ahead of the lower corner. The larger Ca, the more closely the free-surface curve follows the topographic profile. In this case, all curves shows that the film thickness decreases at the left edge of the substrate profile and increases at its right edge. The calculation results are in good agreement with the data of [2].

Next, it is necessary to elucidate whether, in the model considered, it is possible to reduce capillary waves (or even to eliminate them) by means of thermal action, using the topographical features of the substrate. Kabova and Kuznetsov studied [4] a fluid flow with variable viscosity over a two-dimensional substrate surface in the lubrication approximation. It was shown that in the case of fluid flow onto the heated substrate or local heating of the substrate,

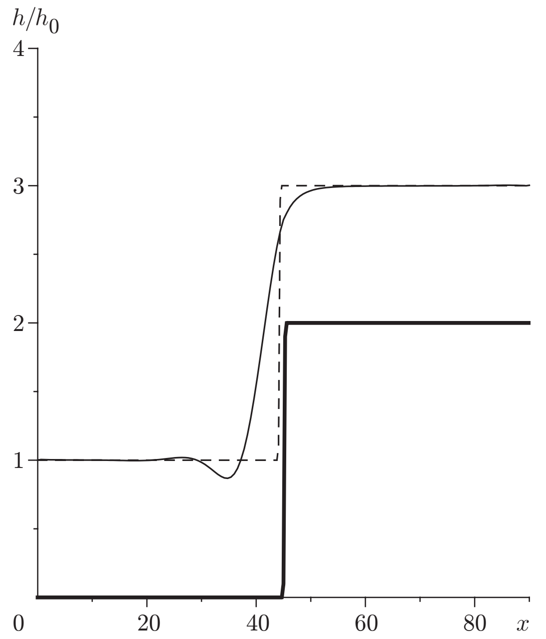

Fig. 3

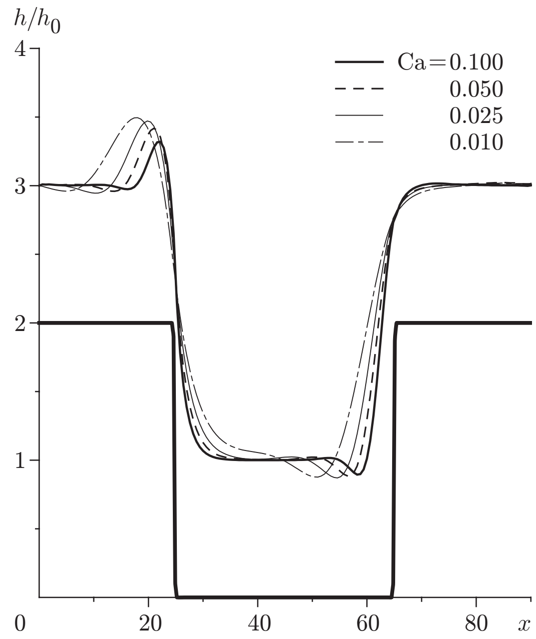

Fig. 4

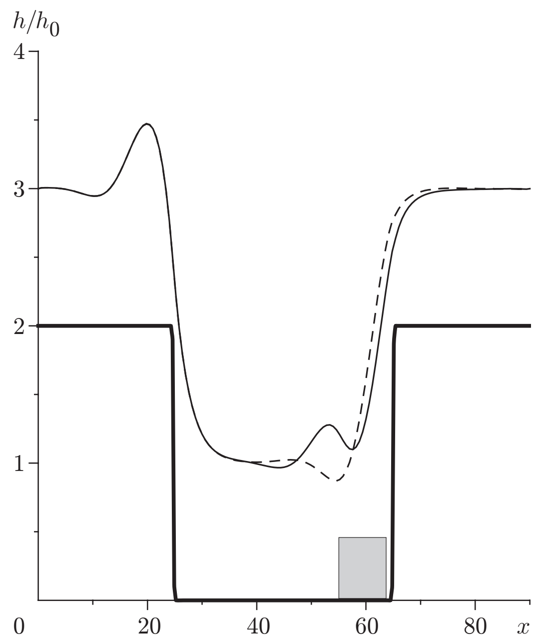

Fig. 5

complex structures of the capillary wave type — fluid rolls — form, which are followed by thinning of the film. These structures also arise in the case of constant viscosity. A similar situation is presented in Fig. 5, where the dashed curve shows the initial location of the free surface (the steady-state solution obtained for an isothermal film flow at Ca = 0.025) and the solid curve is the steady-state solution for Gr = 0.5, Mn = 10, Pr = 1, and a local substrate heating given by the function

$$\theta = 0.45, \qquad 55.8 \leqslant x \leqslant 64.8.$$

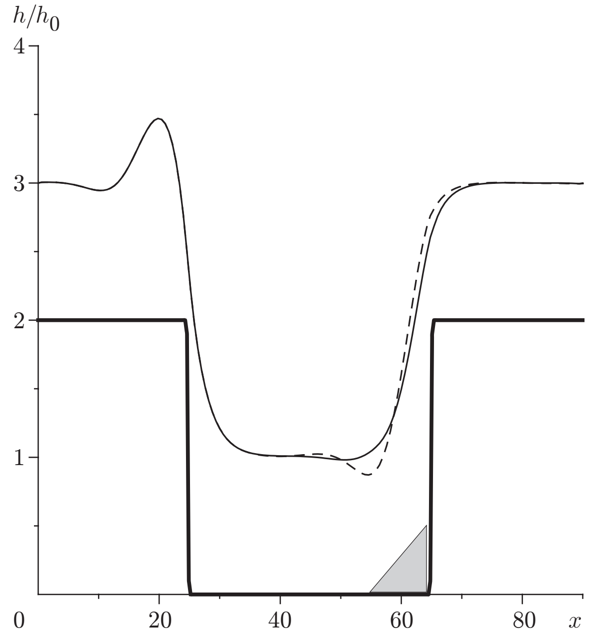

Fig. 6

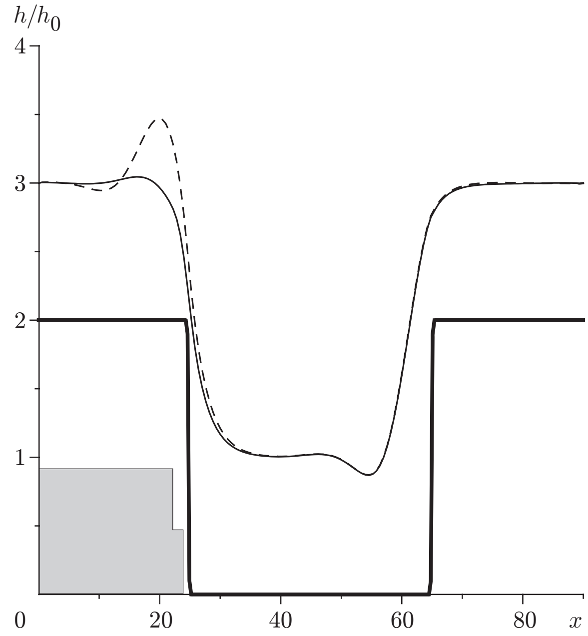

Fig. 7

At the remaining points, $\theta = 0$. For the free-surface temperature, the condition $\partial\theta/\partial n = 0$ is specified. In Fig. 5, the position of the heater and the temperature profile are shown by hatching. The local heating leads to the formation of a fluid roll, which, according to [4] should be followed thinning of the film, but the topographical profile is such that the film moves along the edge of the trench.

Figure 6 illustrates that changing the temperature function, for example, by specifying it as

$$\theta = 0.05(x - 55.8), \qquad 55.8 \leqslant x \leqslant 64.8,$$

one can achieve that the depression ahead of the right edge of the trench disappears and the film flow over the topography becomes more uniform. We note that the left-hand capillary ridge remains unchanged in both cases.

Figure 7 gives the steady-state solution obtained for Gr $= 0$, Mn $= 10$, and Pr $= 1$. The dashed curve shows the initial location of the free surface, and the solid curve is the steady-state solution. The substrate temperature function has the form

$$\theta = \begin{cases} 0.9, & 0 \leqslant x \leqslant 22.5; \\ 0.48, & 22.5 < x \leqslant 24.6. \end{cases}$$

At the remaining points, $\theta = 0$. For the free-surface temperature, the condition $\partial\theta/\partial n = 0$ is specified. The position of the heat source and the temperature profile are shown by hatching.

From the results obtained (see Figs. 6 and 7), it follows that one can significantly influence the free-surface profile of the film on its separate segments can be (for example, decrease the capillary-wave amplitude) by changing the substrate temperature distribution.

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